U.G. 4th Semester Examination - 2020

PHYSICS

[HONOURS]

Course Code : PHYH-CC-T-8

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP-A

1. Answer any **five** questions: 2×5=10

a) Find the roots of $\sqrt[3]{i}$.

b) Evaluate $\tanh\left(\frac{i\pi}{4}\right)$.

c) State whether the function $|z|^2$ satisfy the Cauchy-Riemann condition.

d) If $f(z)=1+iz$, is $\overline{f(z)}=f(\overline{z})$?

e) Find the Fourier transform of $\delta(t)$?

f) If $g(\omega)$ is Fourier transform of f(x), show that $g(-\omega)=g^*(\omega)$ is a necessary and sufficient condition for f(x) to be real.

g) Find the Laplace transform of the function $\delta(t-t_0)$.

h) If f(s) is the Laplace transform of f(t) then find the Laplace transform of the function f(at).

GROUP-B

2. Answer any **two** questions from the following: 5×2=10

a) Prove that if $f(z)=u+iv$ is analytic in a region, then u and v satisfy Laplace's equation in the region.

b) Find the Cauchy-Riemann equations in polar coordinates.

c) Find the Fourier transform of the finite wave train

$$f(t)=\begin{cases}\cos\omega_0 t & |t|\le a\\ 0 & |t|>a\end{cases}$$

d) Find the inverse Laplace transform of $f(s)=\dfrac{6}{(s^2+9)^2}$.

GROUP-C

Answer any **two** questions from the following: 10×2=20

3. a) If C is a circle of radius ρ about z_0, show that

$$\oint_C \frac{dz}{(z-z_0)^n} = 2\pi i \text{ if } n=1$$ but for any other integral value of n, positive or negative, the integral is zero.

b) Find the Laurent series of the function

$$f(z)=\frac{1}{z(z-1)(z-2)}$$ for each annular region between singular points. 5+5

4. a) Evaluate the integral

$$\oint_C \frac{\sin 2z\,dz}{(6z-\pi)^3}$$

where C is the circle $|z| = 3$.

b) Find the indefinite integral

$$\int_{-\infty}^{\infty} \frac{dx}{x^2+a^2}$$

by contour integration. 5+5

5. a) Find the Fourier transform of the normalised Gaussian distribution

$$f(t)=\frac{1}{a\sqrt{2\pi}}\exp\left(-\frac{t^2}{2a^2}\right), \quad -\infty < t < \infty .$$

b) Solve the one-dimensional heat flow equation

$$\frac{\partial \psi}{\partial t} = \kappa^2 \frac{\partial^2 \psi}{\partial^2 x}$$

using Fourier transform where the solution $\psi(x,t)$ is the temperature at position x and time t. 5+5

6. a) Solve the initial value problem

$$\frac{d^2y}{dt^2}+9y=2\sin 3t,$$

using Laplace transform when $y(0) = 1$ and $y'(0)=0$.

b) Find the inverse Laplace transform of the function

$$f(s)=\frac{s^2}{(s^2+a^2)(s^2+b^2)}$$

for $a^2 \neq b^2$. 5+5